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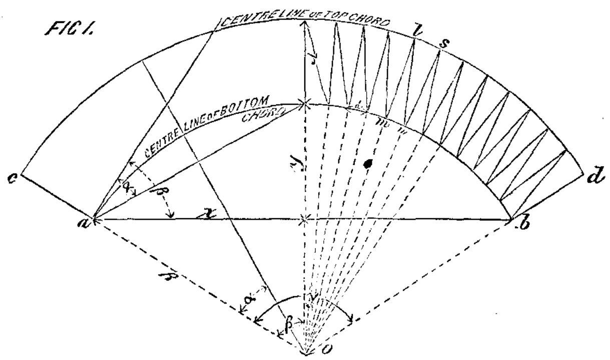
Camber in Bridges. By ALFRED P. BOLLER, C.E.

The following investigation of the question of camber in bridges may be of interest and use to the younger members of the profession, and in that hope and spirit I submit it. In a horizontal truss, composed of top and bottom chords, diagonals, and verticals, the panels formed by the verticals are rectangular, the diagonals being simply the diagonals of rectangular figures. But where the beam is curved, or has a camber introduced, (which is measured by the versed sine of the curve at the centre of the chord line,) these verticals become the radii of a circle, of which the curved chords are arcs, and the diagonals are lengthened.

FIG. 1.



Take, as in the figure, an exaggerated case, letting $a\,b$ and $c\,d$ represent the arcs formed by the bottom and top chords, respectively, and also let n equal the number of panels, d being their distance from centre to centre, measured along the centre line of the bottom chord.

Let γ = depth of truss, or rather, the distance between the centres of gravity of the top and bottom chords.

y and x = the camber and half span.

R = radius of the arc $a\,b$.

β, γ, θ = angles as marked on diagram.

n, d, x, and y are given, also $a\,b$ equal $n\,d$.

The trussed beam being built in a curve, the upper chord must, of course, be lengthened by an amount proportional to the radius of curvature, which produces a change in the panels, causing them to be longer along the top chord than along the bottom chord. The amount of this increased length is, of course, the same for every panel. The length of the top chord is represented by $\frac{\pi \rho v}{180} = 0{\cdot}0175\ (\mathrm{R} + r)\ v$, in which $v = \gamma$. Therefore the total amount increase in the length of the top chord over the bottom chord is

$$0{\cdot}0175\ (\mathrm{R} + r)\ v - n\,d = \delta, \quad . \quad . \quad . \qquad (1.)$$

To find the radius of curvature, R, we have

$$\tan\ \alpha = \frac{y}{x};\ \beta = 2\,\alpha\ ;\ \Upsilon = 4\,\alpha\ ;\ \text{and}\ \theta = \frac{\gamma}{n}, \quad . \qquad (2,)$$

$$\sin\beta = \frac{x}{\mathrm{R}},\ \text{and}\ \mathrm{R} = \frac{x}{\sin\beta}, \quad . \quad . \qquad (3.)$$

Now, the number of panels being n, and the amount of *total* increase being δ, the lengthening for each panel along the top chord is—

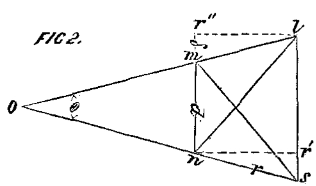
$$\frac{n}{\delta} \quad . \quad . \quad . \quad . \qquad (4.)$$

To find the length of the diagonals, let (Fig. 2) represent one panel, O being the centre of curvature. We have $r'\,n\,\mathrm{s} = \frac{1}{2}\,\theta = r''\,l\,m$; $r''\ m = \frac{n}{2\,\delta}$, $\cos\frac{1}{2}\,\theta = \frac{l\,r''}{m\ l}$, and $l\,r'' = \cos\frac{1}{2}\,\theta \times m\ l$.

Then, from the right angled triangle $l\,r''\,n$, there results—

$$ln = \sqrt{r''\,l^2 + r''\,n^2}, \quad . \qquad (5,)$$

FIG. 2.



which is the length of diagonal.

Take as an example the camber in the proposed wrought iron International Bridge, to be built at Buffalo, over the Niagara River, which is intended to be six inches. r is 20·75 feet, the number of panels 24, and 10·7 feet, from centre to centre, along centre line of bottom chord.

$$a\,b = 10{\cdot}7 \times 24 = 256{\cdot}8.$$

$$\tan\ \alpha = \frac{y}{x} = \frac{{\cdot}50}{128} = {\cdot}003907 = 0^\circ - 13' - 30''.$$

$$\beta = 0^\circ - 27'\,;\ \gamma = 0^\circ - 54',\ \text{and}\ \theta = 0^\circ - 2' - 14''.$$

$$\text{Sin}\ \beta = \frac{x}{\text{R}},\ \text{and}\ \text{R} = \frac{x}{\sin \beta} = \frac{128}{\sin 27'} = \frac{128}{\cdot 00785} = 16{,}305\cdot 7\ \text{feet.}$$

Length of $c\,d$,

$$0\cdot 0175\ (\text{R} + r)\ v = \cdot 0175 \times 16{,}326\cdot 4 \times \tfrac{9}{10}{}^\circ = 257\cdot 13,$$

$$c\,d - a\,b = 257\cdot 13 - 256\cdot 8 = 0\cdot 33\ \text{feet} = 4\ \text{inches.}$$

The amount that each panel is to be lengthened along the top chord is $\frac{4}{24} = \frac{1}{6}$ of an inch.

For the lengths of the diagonals we have—

$$r'\,n\,\text{S} = \tfrac{1}{2}\,\theta = 0^\circ - 1' - 07''\,;\ m\,n = 10\cdot 70\,;\ n\,\text{S} = 20\cdot 75.$$

$$r''\,m = \cdot 0075\,;\ \cos 0^\circ - 1' - 07'' = \frac{l\,r}{m\,l},\ \text{and}\ l\,r = 20\cdot 75 \times 1 = 20\cdot 75$$

feet.

$$l\,n = \sqrt{r''\,l^2 + r''\,n^2} = \sqrt{545\cdot 21} = 23\cdot 35\ \text{feet.}$$

The practice among many excellent bridge mechanics is to distribute equally the amount of camber among the panels along the top chord, which always increases the amount of camber. For example: Instead of distributing equally *four* inches among the panels, found in the above case to be required for a six-inch camber, they would distribute *six* inches, which would give nearly one-third more camber.

When camber is properly constructed in a bridge, the bottom chords are not strained by tension until the deflection of the truss is so great as to pass the horizontal chord-line. *Until* then the whole truss acts like a flat arch, and consequently, when the camber is considerable, no deduction for loss of strength by area cut away need be provided for, as would be necessary for a straight built beam, when deflection to the slightest amount would call upon the bottom chord to resist tension.

The graphical construction of the camber, in designing a bridge truss, may be very readily accomplished by starting from the *centre vertical*, working both ways to the abutments. First measure off the distance between centres of gravity of sections of the top and bottom

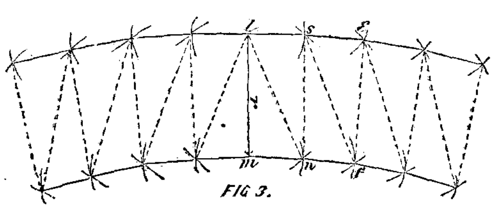


FIG 3.

chord along the centre vertical. Thus, in Fig. 3, $l\,m$ is the vertical at centre $= r$ in formula; $m\,n = d$; $l\,s = d + \frac{n}{\delta}$, and $l\,n$ is the calculated diagonal.

From the point m, with radius d, draw an arc, and with l as a centre, and the diagonal as the radius, cut this arc in n. With l as a centre, and $d + \frac{n}{\delta}$ as a radius, describe an arc, cutting it in s, using the diagonal as radius, with m as a centre. Repeat this same operation from the points n and s, to get the points e and f, and so on to the abutments. This method can be relied upon as simple, rapid, and accurate.

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Sextant with Attached Spirit-level Horizon. By George Davidson, Assistant U. S. Coast Survey.

It very frequently happens that the traveler and the navigator are placed in situations where it is in the highest degree desirable to determine their geographical positions, or to measure the elevation of some mountain by an altitude or depression, and yet the means at their command may be inefficient. At sea, the sextant is the best available instrument for measuring angles of elevation or azimuth, but for the former purpose it can be used only when the object and sea horizon are both visible. Sometimes, however, when it is very desirable to measure an altitude, the sun is so low that the bright and dazzling reflection from the surface of the water obscures the horizon; or the horizon is hidden by a low fog whilst the sun is visible through it.

On shore, the traveler, whose outfit of astronomical instruments is limited to a pocket chronometer, sextant, and artificial horizon, finds constantly annoying occasions when his means fail him in important determinations. The sun may be too high for observing double reflections with the ordinary sextant; the sun or a mountain may be too low to admit of available reflection in the artificial horizon; and in particular these means positively fail him when, from any elevated point, he wishes to measure the depression of some object, as of the sea horizon, by which to determine his elevation or distance, knowing one of them, or, knowing the distance of another and lower mountain, to determine the difference of elevation.

We have encountered all these difficulties, and also the less frequent one of falling in with a reef at night with the stars visible but the sea horizon totally obscured in darkness. This occurrence first directed our attention to the subject of adjusting an artificial horizon to the sextant about twelve years since, but we failed to solve the problem. Three years since, when daily using the hand-level generally known to the trade as Locke's level, we saw the means at our hand ready for application, and fitted it to a Gambey sextant.

Plate IV. shows this level in detail. Fig. 1 gives the general appearance of the sextant and the attached level; Fig. 2 is a transverse section through the bubble, cross wire, and reflector; Fig. 3 is a longitudinal section through the bubble, reflector, and double convex lens. The tube B carrying the spirit-level is closed at each end with